ONE PROBLEM OF CONTROLLING THE HEATING OF CYLINDRICAL PELLETS

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The problem of controlling the heating of cylindrical pellets was formulated and solved on a computer with the aid of a mathematical experiment. On the basis of the obtained results recommendations for some design parameters and the intensity of the heating current are given.

1. It is known that a temperature gradient in a compressed preform of powdered material leads to harmful effects. Inhomogeneity of the temperature field manifests itself particularly strongly in the compaction of powdered alloys whose thermal conductivity is low (e.g., titanium alloys). In that case cooling proceeds along the periphery of the pellet, and when it is deformed, a coarsely crystalline ring with reduced strength properties may form. Inhomogeneity of deformation in compacting, arising on account of inhomogeneity of the temperature field, leads to the appearance of discontinuities of the surface of components. Thus the quality and the mechanical properties of a product are directly dependent on the homogeneity of the temperature field during the process of sintering.

One of the methods of heating metallic powder placed in a cylindrical mold in hot molding is heating it by a harmonic current with frequency ω and amplitude I [1]. For that purpose the pellet is placed between electrodes whose shape and dimensions may be variegated.

One of the problems of controlling the sintering process is the evaluation of the current intensity I for the specified characteristic frequency of the current and with typical values of the parameters of heat exchange with the environment, and the choice of shapes and dimensions of the electrodes from a set of possible shapes and dimensions so that in steady regime uniform heating, within the tolerance δ, of the pellets to the required temperature $\hat{u}$ is attained.

2. The mathematical statement reduces to the problem of "quasiminimization" of the object functional $\Phi(I, p)$, where p is the set of parameters characterizing the shape and dimensions of the electrodes of the given set P:

$$\Phi(I, p) \leqslant \delta, \ p \in P, \ 0 \leqslant I \leqslant \bar{I}. \tag{1}$$

If a cylindrical pellet occupying the domain $V \equiv \{M(r, z, \varphi) \ E_3: \ 0 \leqq r \leqq R, -\ell \leqq z \leqq \ell, 0 \leqq \varphi \leqq 2\pi\}$ is heated, then it is natural to choose as object functional

$$\Phi(I, p) = \max_{M \in V} |u(M, I, p) - \hat{u}|, \tag{2}$$

where u(M, I, p) is the temperature at the point M algorithmically determined (see Sec. 3) in the regime of quasisteady heating at each pair (I, p).

If p is chosen from a small number of possible values, and since the dimensions of the electrodes are deliberately limited by the dimension R, and $I \in [0, \bar{I}] \subset E_1$, the problem (1)-(2) is solved on some compact $Z = P \times [0, \bar{I}]$ and turns out to be posed correctly according to Tikhonov with the specified tolerance δ [2] if $\delta \geqq \delta_0 = \inf_Z \Phi(I, p)$. In that case, it can be solved by the method of selection on an a priori specified set Z, which is also done below.

3. On account of the nonlinearity of the corresponding problem, a special complication is presented by the calculation of the temperature of a body u(M, I, p) determined by the following system:

$$A(u, H) \equiv \frac{1}{r} \frac{\partial}{\partial r}\left(rk(u) \frac{\partial u}{\partial r}\right) + \frac{\partial}{\partial z}\left(k(u) \frac{\partial u}{\partial z}\right) + \frac{\lambda(u)}{2}\left(\left|\frac{\partial H}{\partial z}\right|^2 + \left|\frac{1}{r} \frac{\partial}{\partial r}(rH)\right|^2\right) = 0, \ M(r, z, \varphi) \in V, \tag{3}$$

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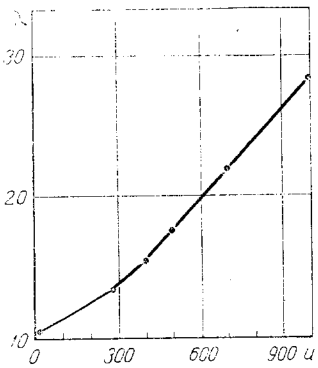

Fig. 1

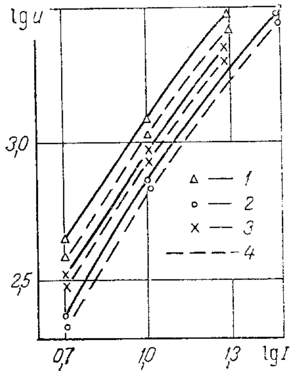

Fig. 2

Fig. 1. Experimental dependence k = k(u). k, W/m·K; u, °C.

Fig. 2. Dependence of the temperature on the current: disk electrodes: 1) $a = R/4$; 2) $a = R/2$; annular electrodes: 3) $a = R/4$; $b = R/\sqrt{8}$ (1-3) for the maximal; 4) for the minimal temperature level.

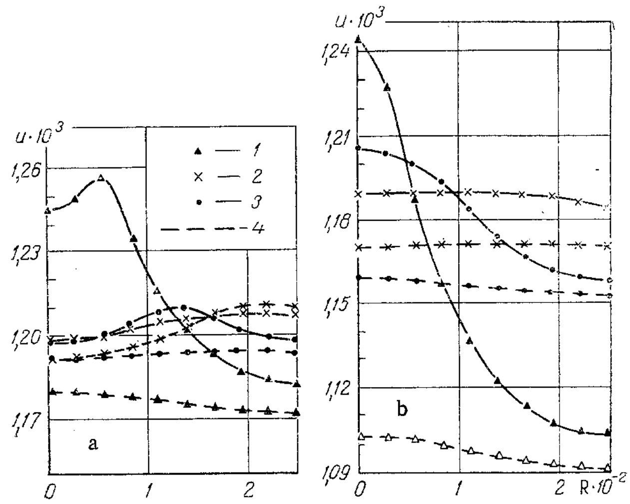

Fig. 3

Fig. 3. Dependences of the temperature field in the optimal regime on the radius for annular (a) and disk (b) electrodes: a. 1) $a = R/4$, $b = R/\sqrt{8}$; 2) $a = R/\sqrt{2}$, $b = R$; 3) $a = R/2$, $b = R/\sqrt{2}$ (1-3, for the maximal, 4, for the minimal temperature level); b. 1) $a = R/4$; 2) $a = R/\sqrt{2}$; 3) $a = R/2$. u, °C, R, m.

$$-k(u)\left.\frac{\partial u}{\partial r}\right|_{r=R} = \alpha\left(u|_{r=R} - u_0\right), \quad -k(u)\left.\frac{\partial u}{\partial z}\right|_{z=\pm l} = \pm\alpha\left(u|_{z=\pm l} - u_0\right), \tag{3}$$

$$\lim_{r\to 0} rk(u)\frac{\partial u}{\partial r} = 0,$$

$$B(u, H) \equiv \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{1}{r\lambda(u)}\frac{\partial}{\partial r}(rH)\right) + \frac{\partial}{\partial z}\left(\frac{1}{\lambda(u)}\frac{\partial H}{\partial z}\right) + i\omega\mu(u)H = 0,$$

$$H|_{r=0} = 0, \quad H|_{r=R} = \frac{I}{2\pi R}, \quad H|_{z=\pm l} = \frac{I}{2\pi r}f(r),$$

TABLE 1. Principal Characteristics of the Process in Dependence on the Electrode Design

Process Parameter	Current amplitude I_{opt}, A	Estimate of uniformity Δ, °C	Current density $j \cdot 10^4$, A/m²
Variant 1:			
disk $a = R/\sqrt{2}$	15,5	20	1,58
ring $a = R/\sqrt{2}$, $b = R$	18	22	1,83
Variant 2:			
disk $a = R/2$	15	49	3,06
ring $a = R/2$, $b = R/\sqrt{2}$	17,5	21	3,58
Variant 3:			
disk $a = R/4$	10,5	156	0,85
ring $a = R/4$, $b = R/\sqrt{8}$	13	85	1,06

where for the axisymmetric electrodes f(r) is their shape factor that also includes dimensions.

To solve this problem, we devised a control unit program based on the conservative difference schema [3] and solved by Seidel's method. The difference schema is realized within the framework of the iteration method:

$$A(u,\ H^{(s)}) = 0 \Rightarrow u^{(s)},$$
$$B(u^{(s)},\ H) = 0 \Rightarrow H^{(s+1)},\quad s = 0, 1, \ldots,$$

where the initial approximation is $u^{(0)} = u_0$.

4. The results of the calculations presented below were obtained for the following invariant parameters characteristic of the structure under consideration: ω = 1 kHz, heat transfer coefficient at the boundary α = 15 W/m²·deg, the dependence k(u) is presented in Fig. 1, $\lambda = \lambda(u)$ is correlated with k by the Wiedemann–Franz law: $k/\lambda = TL_0$, where T is the Kelvin temperature, $L_0 = 2.44 \cdot 10^{-8}$ W·Ω/K². Density and heat capacity of the material were taken, respectively, $\rho = 8.3 \cdot 10^3$ kg/m³, c = 0.55 kJ/kg·K. The geometric dimensions of a pellet are: R = ℓ = 0.025 m, the required temperature level $\hat{u}$ is 1200°C. In the calculations we compared disk electrodes $0 \leqq r \leqq a$ and annular electrodes $a \leqq r \leqq b$ with equal area so that in the variants to be compared, equal current density j is ensured with specified I. For disk and annular electrodes f(r) is given, respectively, by the following formulas:

$$f(r) = \begin{cases} \dfrac{r^2}{a^2}, & 0 \leqslant r \leqslant a, \\ 1, & a \leqslant r \leqslant R, \end{cases} \qquad f(r) = \begin{cases} 0, & 0 \leqslant r < a, \\ \dfrac{r^2 - a^2}{b^2 - a^2}, & a \leqslant r \leqslant b, \\ 1, & b < r \leqslant R. \end{cases}$$

The parameters of the electrodes in the compared variants are presented in Table 1 together with the summary results of the calculations.

Figure 2 presents the dependences of the regionally maximal and minimal temperature on the amplitude of the harmonic current for different variants of electrode configurations and the optimal values of current at which, "on an average," the lowest temperature level is attained. These values of I_{opt} are also presented in Table 1. The graphs of the figure may be regarded as nomographs for choosing the optimal regimes; this apparently admits of full automation.

Figure 3 shows the dependences of the temperature field on the radius with two fixed z at which in the optimal regime ($I = I_{opt}$) the temperature assumes the maximal and minimal values. These dependences are also presented in the figure for different electrode configurations with an indication of the estimate of the deviation Δ from the mean value (see also Table 1).

5. The obtained results permit the following recommendations to be made.

If the outer radius of the ring b is not very close to R, then the annular shape of the electrodes ensures a more uniform temperature field than the disk shape. This result corresponds to the qualitative evaluations obtained in the study of the current density distribution for operation with dc. The "anomalous" effect when b ~ R is due to the effect of heat transfer from the surface, which in our case is considerable.

Out of the examined annular electrodes, the greatest "uniformity" with optimal current is ensured by variant 2 which corresponds to the "mean position" of the ring: $a = R/2$, $b = R/\sqrt{2}$.

Table 1 also presents the current densities on the electrodes. It can be seen that the current density competes with the level of uniformity of the field, and in the variant that is optimal from the point of view of uniformity of the field the current density is greatest although it does not exceed the permissible values.

LITERATURE CITED

1. M. S. Kovalenko, The Theoretical Foundations of Hot Forming of Porous Materials [in Russian], Naukova Dumka, Kiev (1980).
2. A. N. Tikhonov and V. Ya. Arsenin, Methods of Solving Malposed Problems [in Russian], Nauka, Moscow (1979).
3. A. A. Samarskii, The Theory of Difference Schemata [in Russian], Nauka, Moscow (1977).

REGULAR REGIME IN TRANSLUCENT MATERIALS

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The article analyzes radiative—conductive heat transfer in a translucent plate. It is established that a regime exists which is analogous to the regular regime in conductive heat transfer.

The question of the existence of regular regimes in translucent materials was dealt with by several authors [1-4] but the results of [1, 2] differed substantially from those of [3, 4]. The authors of the last two articles explained this difference by stating that in [1, 2] the conductive component of heat transfer was predominant whereas in [3, 4] it was the radiative component. However, the authors of [1, 2] investigated the regular regime with radiative and convective heat transfer acting in the same direction (radiative and convective heating or radiative and convective cooling) whereas in [3, 4] these components were opposed to each other (convective heating and radiative cooling). This required additional theoretical investigation of the regular regime in a translucent plate where the radiative component of heat transfer is equal to or larger than the conductive component, and both act in the same direction.

The difference algorithm for investigating nonsteady radiative—conductive heat exchange was constructed in the following manner: the energy equation for an infinitely thin layer of a translucent plate with optically smooth surfaces, separated from the opaque surfaces by a medium conducting thermal radiation, can be expressed in the form [5]

$$C_\gamma \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[\Lambda \frac{\partial T}{\partial x}\right] - \frac{\partial q^r}{\partial x}, \quad 0 \leqslant x \leqslant b, \tag{1}$$

where $\partial q^r/\partial x$ is determined by the expression

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